NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science (Hons) in Applied Mathematics			
QUALIFICATION CODE:	08BSHM	**LEVEL:**	8
COURSE CODE:	ADC801S	**COURSE NAME:**	ADVANCED CALCULUS
SESSION:	JUNE 2022	**PAPER:**	THEORY
DURATION:	3 HOURS	**MARKS:**	100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER:	DR. DSI IIYAMBO
MODERATOR:	PROF. OD MAKINDE

INSTRUCTIONS

1. Attempt all the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Question 1.

Suppose that the equation $xe^{yz} - 2ye^{xz} + 3ze^{xy} = 1$ defines z as an implicit function of x and y. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

[10]

Question 2.

Find the local extreme values and the saddle points of the function $f(x, y) = 4 + x^3 + y^3 - 3xy$.

[14]

Question 3.

Use the method of *Lagrange multipliers* to find the minimum and maximum values of the function $f(x, y) = 2x^2 + y^2 + 2$, where x and y lie on the ellipse C given by $x^2 + 4y^2 - 4 = 0$. [15]

Question 4.

Let $\mathbf{F} = (e^x \ln y)\mathbf{i} + \left(\frac{e^x}{y} + \sin z\right)\mathbf{j} + (y \cos z)\mathbf{k}$.

a) Determine whether $\mathbf{F}$ is a conservative vector field. If it is, find a potential function for $\mathbf{F}$.

b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 5\mathbf{k}$, where $0 \leq t \leq 2\pi$.

[19,5]

Question 5.

Let f be a differentiable function of x, y and z, and let $\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$, where P, Q and R are differentiable functions of x, y and z. Prove that $\text{div}(f\mathbf{F}) = f\text{div}\mathbf{F} + \mathbf{F} \cdot \nabla \mathbf{f}$.

[10]

Question 6.

Evaluate $\int_C xy^4 \, dS$, where C is the upper half of the circle $x^2 + y^2 = 16$ in the counter clockwise direction. [9]

Question 7.

Use Green's Theorem to evaluate $\oint_C y^3 \, dx - x^3 \, dy$, where C is the positively oriented circle of radius 2 centred at the origin. [10]

Question 8.

Evaluate the integral $\iiint_B 8xyz \, dV$ over the box $B = [2, 3] \times [1, 2] \times [0, 1]$. [8]